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Fermi problems as content, connectors and integrators in mathematics and between the STEAM disciplines

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Abstract

This paper presents a discussion and analysis of recent developments and investigations into the multifaceted role of Fermi Problems as content, connectors, and integrators in mathematics and across the STEAM disciplines. In particular, the role of Fermi problems as interdisciplinary activities in the context of mathematical modelling and problem posing is discussed, as well as the utility of the framework known as the Fermi Problem Activity Template (FPAT) as a tool for supporting the design and implementation of Fermi problems at different levels. The paper then proceeds to present a number of illustrative examples and recommendations for facilitating the integration of Fermi problems into curriculum materials and classroom practice. Furthermore, it outlines potential avenues for future research in this field.

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1. Introduction

The ability and importance for students to make connections between content and different subject areas in the learning of mathematics and the STEM disciplines has since long been established in research (Cai & Ding, 2017; Murphy, Hatisaru & Chick, 2024; Kelly & Knowles, 2016). Making connections allows students to have a more coherent and nuanced view of mathematics, to better retain the mathematics they have learned, and to use their mathematical knowledge more efficiently (Bossé, 2003; Murphy, Hatisaru & Chick, 2024). Connections within mathematics, *intra-mathematical connections*, "are formed between ideas, concepts, theorems, procedures, or representations within mathematics" (Murphy, Hatisaru & Chick, 2024, p. 231), while connections between mathematics and the other STEAM disciplines are *extra-mathematical connections* between mathematics and contexts, problems, phenomena and situations from outside of mathematics (Gamboa et al., 2021; Niss & Blum, 2020). However, students have difficulties in making such connections (Hatisaru, 2023; Henri Retnawati, Apino & Santoso, 2020) and it is challenging for teachers to support students in making these connections (Businskas, 2008; de Gamboa et al., 2020; Hatisaru, & Erbaş, 2017). Furthermore, the subject of mathematics is at risk of becoming invisible when making extra-mathematical connections, especially in STEM settings (English, 2016; Fitzallen, 2015).

In this paper I discuss how to facilitate students making intra- and extra-mathematical connections using so-called Fermi problems (FPs). After presenting some theoretical background, I define FPs and give a brief summary of educational research focusing on FPs. I then go on to elaborate on how FPs can be used both as content and as connectors and integrators within mathematics and between the STEAM disciplines, giving some examples and outlining future avenues for further research in this area using FPs.

2. Theoretical considerations

2.1 Mathematical modelling: Educational research focusing on mathematical modelling is conducted from a variety of theoretical perspectives and approaches (cf. Kaiser & Sriraman, 2006). Commonly used frameworks in this area of research often draw on a cyclic representation of the modelling process, *a modelling cycle*, or modelling competencies (Cevikbas, Kaiser & Schukajlow, 2022; Niss & Blum, 2020). The various modelling cycles outline different work phases (focusing on aspects such as *the real situation*, *the mental representation of the situation*, *the real model*, *the mathematical model*, *the mathematical result*, and the *real results*) and the transitions between these phases (*understanding the task*, *simplifying/structuring the task*, *mathematising*, *working*

mathematically, *interpreting, and validating*) (Niss & Blum, 2020). In this paper, however, I consider mathematical modelling to be a comprehensive problem-solving approach that involves translating real-world problems, phenomena, and situations into the mathematical domain in order to develop a mathematical model to describe, understand, predict or control some aspect or property of the system being modelled. More generally, mathematical models are conceptual systems that represent other systems (Lesh & Doerr, 2003). These models are composed of a set of concepts designed to explain or describe the mathematical aspects relevant to the object under study. Models also encompass the methods used to create useful constructions, manipulations, or predictions to achieve clearly defined objectives (Lesh & Harel, 2003). It is important to recognise that mathematical modelling is seen as both a content in its own right and as an approach to teach and learn other curricula content (Julie & Mudaly, 2007). A recent line of research in the field of mathematical modelling focuses on problem-posing (Hartmann, Krawitz & Schukajlow, 2023), an area of research that seem particularly promising in the context of research on creativity in the mathematics teaching and learning (Cai et al., forthcoming).

2.2 Problem-posing: In addition to its recent connection to mathematical modelling, the teaching and learning of mathematics through problem-posing is an active area of research in mathematics education that has cumulatively demonstrated the importance, potential and power of using mathematical problem-posing in mathematics classrooms (Cai & Hwang, 2020; Cai & Rott, 2024). For the purposes of this paper, I draw on a definition by Cai and Hwang (2020), which states that "[b]y *problem posing* in mathematics education, we refer to several related types of activity that entail or support teachers and students formulating (or reformulating) and expressing a problem or task based on a particular context (which we refer to as the *problem context* or *problem situation*)" (p. 2), in conjunction with Cai and Rott's (2024) heuristic general problem-posing model stressing *orientation*; *connection*; *generation*; and *reflection* (see Figure 1).

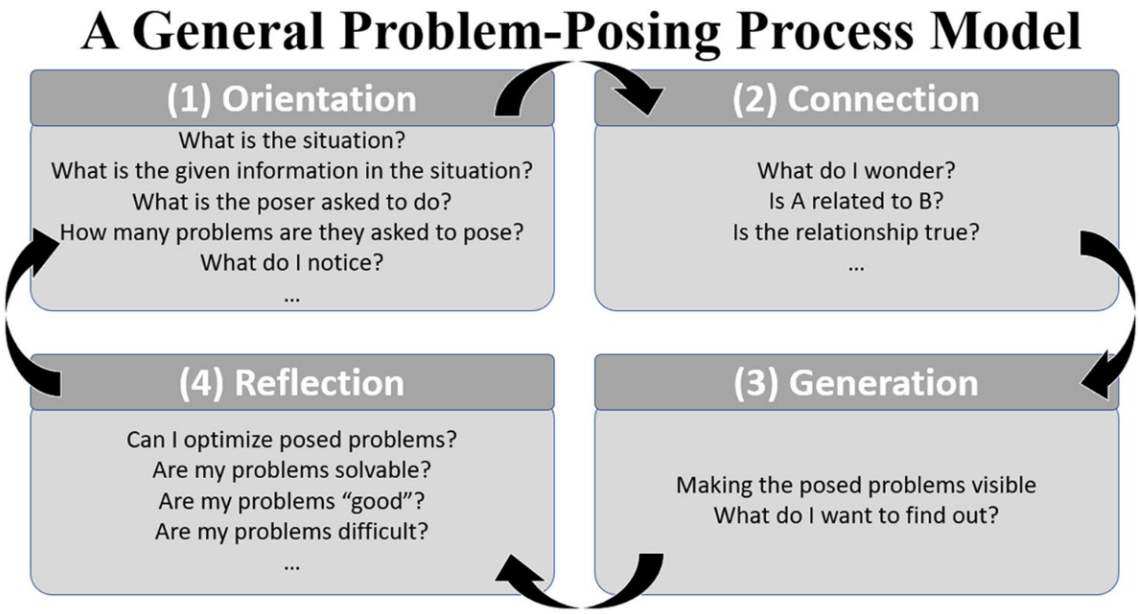

Figure 1. The general Problem-Posing Process Model by Cai & Rott (2024, p. 68)

A strength of this definition of, and the framework for, problem-posing, is that they relaxes the sole focus on the mathematical domain (i.e., not exclusively emphasising *mathematical* problem-posing) and opens up for thinking about problem-posing in other contexts and more diverse domains. This is important for my discussion of the problem-posing aspects when situating FPs in a STEAM setting. Just as problem-posing in modelling has recently been given more attention in the models and modelling discourse, models and modelling have also been stressed as key concepts in STEM education (Hallström & Schönborn, 2019; 2023; Hjalmarson et al., 2020).

2.3 STEAM: Science, Technology, Engineering, Art and Mathematics (STEAM) as a unified educational research discipline has garnered significant interest and prominence in educational research over the past two decades (Perignat &Katz-Buonincontro, 2019). Despite varying interpretations of what STEAM and STEAM education encompass (Fraser et al., 2019; Mejias et al., 2021), Li et al. (2020) argue that a growing consensus and clarity of the terminology in STEM education can be seen in the literature from 2000 to 2018. However, an exception is the 'A' in STEAM – the *Arts* – which still exhibits a variety of interpretations (Perignat &Katz-Buonincontro, 2019). Numerous researchers have pointed out that the role of mathematics within STEAM education often remains obscure (e.g. Choy & Coopere, 2024; English, 2016; Fitzallen, 2015). Rather that promoting the exploration of mathematics in rich interdisciplinary contexts, STEM activities often reduce mathematics to a mere tool for other disciplines (Maass et al., 2019; Shaughnessy, 2013). As part of addressing this challenge, Ärlebäck and Albarracín

(2019) argued that so-called Fermi problems (FPs) could be used both as facilitators for learning in the STEM disciplines (mathematics being no exception) and as integrators between the STEM disciplines.

2.4 Fermi problems: *How many bricks are there in the Great Wall of China? How much rice is consumed annually in China?* These questions are examples of so-called *Fermi problems (FPs)*, which can be tackled by breaking them down into simpler sub-problems. These sub-problems can then be solved using rough estimates of the relevant quantities and straightforward calculations, which together provides the solution to the original problem. This strategy to solve the problem is called the *Fermi (estimate) method* (see Figure 2) coined after the Italian born physicist Enrico Fermi, who introduced and used this type of problems to show the power of deductive thinking, often in the context of preparing his students for experimental laboratory work (Efthimiou & Llewellyn, 2006).

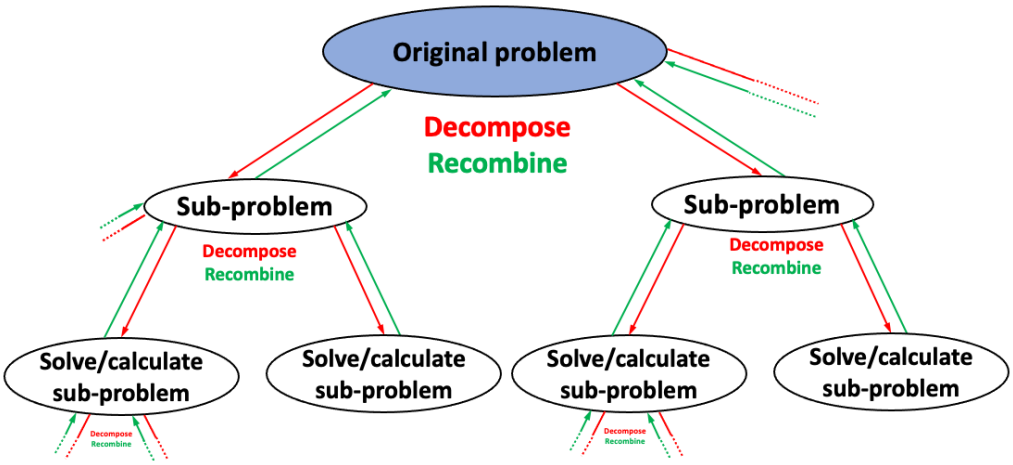

Figure 2. The Fermi (estimate) method

Generally, one can define FPs as "open, non-standard problems requiring students to make assumptions about the problem situation and estimate relevant quantities before engaging in, often, simple calculations" (Ärlebäck, 2009, 331-332). Efthimiou and Llewellyn (2006) describe FPs to have a distinct characteristic in that "they always seem vague with very little or no information given, but they admit dissection into a set of simpler questions that lead to the final answer" (p. 254). The process of decomposing the original problem is similar to engaging in a modeling process (Ärlebäck, 2009), and FPs have been described as miniature modelling problems since they generally are are smaller, more well-defined, and more focused real-world problems (Robinson, 2008).

Although originating in physics education, FP have been used with an increasing interest in mathematics education during the last two decades with many diverse foci. Based on the past years cumulative research this lead Ärlebäck and Albarracín (2023) to propose FPs as a hub for task design in mathematics and STEM education.

3. Research on Fermi problems – in mathematics and STEAM

The use of FPs for various educational purposes has bee shown to be feasible, since the Fermi method has been found to be appreciated by students and a method that can successfully be taught to students (Barahmeh, Hamad & Barahmeh, 2017). Indeed, looking across the STEM disciplines, the literature review by Ärlebäck and Albarracín (2019) shows that FPs have been implemented in all STEM disciplines. However, since a comprehensive review of the literature on FPs is beyond the scope of this paper, I will provide a brief summary of selected research to discuss Femi problems as connectors and integrators of content, and the connections between FPs and problem-posing. In turn, I will focus on the research in mathematics (intra-mathematical connections) and then STEAM more generally (extra-mathematical connections).

3.1 Fermi problem as intra-mathematical connectors and integrators

The research on FP in an intra-mathematical setting can broadly be divided into two categories depending on if FPs is seen as a content in its own right or as a vehicle to learn other mathematical content (cf. Julie& Mudaly, 2007). An interesting line of research that simultaneous involves both these categories is investigating how sequences of FPs can help students develop more complex models adaptable to new situations, which both enhance students modelling abilities as well as deepen their mathematical knowledge about the content the sequence in question is designed to focus on (Albarracín & Gorgorió, 2018; Ferrando & Albarracín, 2021; Albarracín et al., 2022).

3.1.1 Fermi problems as content: When FPs are thought of as content in mathematics education research, they are fundamentally seen as instantiations of modelling problems (cf. Peter-Koop, 2009; Ärlebäck, 2009). As such, the research shows how students working on FPs are able to engage in mathematical modelling in productive ways at all school levels (see Albarracín et al. (in review) for a more detailed overview). First of all, FPs have been used successfully to introduce mathematical modelling at the primary level (Albarracín & Gorgorió, 2019; Peter-Koop, 2009) as well at the upper secondary level (Ärlebäck, 2009), where at the at the primary level students are

able to generate own mathematical models to solve problems (Albarracín, 2021). At the upper secondary level, student have a greater capacity to in-cooperate their experiences and knowledge from other real-world contexts in the modelling process (Ärlebäck, 2009), and producing more complex models (Albarracín & Gorgorió, 2014). At the tertiary level, FPs are found to prompt students' higher meta-cognitive reflection and awareness (Czocher, 2016, 2018). In addition, multiple studies have shown how sequences of FPs can support students in developing and adapting successively more complex models (Albarracín & Gorgorió, 2018; Ferrando & Albarracín, 2021; Albarracín et al., 2022).

The research summarised above on FPs as content in relation to modelling activities focuses on the process of modelling (cf. Niss & Blum, 2020). In this sense, the emphasis is on developing ways of solving FPs and supporting students in acquiring the necessary mindsets and skills. This can be seen as asking students to develop their metacognitive awareness and skills, which in the context of problem posing as understood in this paper (cf. Cai & Hwang, 2020; Cai & Rott, 2024), is about learning the General Problem-Posing Process Model (Cai & Rott, 2024). When engaged in solving FPs, or mathematical modelling in general, from a metacognitive perspective, meta-questions analogue to the four phases of *orientation*, *connection*, *generation* and *reflection* (see Figure 1) could support students in developing the necessary mindsets and metacognitive skills to become proficient modellers: 'Where am I in the modelling process right now?' (*orientation*); 'Which phase of the modelling process would be most productive to engage in next?' (*connection*); 'What can I do, and how can I document the modelling processes I am engaged in?' (*generation*); and 'What am I/have I learned about solving modelling problems to facilitate solving the next modelling problem?' (*reflection*).

3.1.2 Fermi problems as a vehicle for learning other mathematical content: Although FPs can come across as unfocused and open with respect to specific content, they do connect to relevant mathematical connect and skills, (Carlsson, 1997; Ärlebäck, 2009). More often than not, higher level abilities and skills and competency goals (modelling excluded here) are stressed as the learning goal in the research involving FPs. Some examples are: *number sense* focusing on large number (Albarracín, 2021); *estimation* (Albarracín, 2021; Albarracín & Gorgorió, 2019); *(general) mathematical reasoning skills* (Albarracín, 2021; Albarracín & Gorgorió, 2019); and *flexible problem-solving skills* (Segura & Ferrando, 2023; Segura et al., 2023). Recently, and related to curricula goals aimed at foster creativity in students' mathematical work (cf. Cai et al., forthcoming), research

indicate a clear relationship between success in solving FPs and mathematical creativity at the secondary level (Okamoto, 2022; Okamoto, et al., 2023).

A more delimited content and fundamental mathematical idea inherent in, at least in most initial, solutions to FPs is *proportionality* (Carlsson, 1997; Peter-Koop, 2009; Ärlebäck, 2009). Another example of a content-focused use of FPs, is the sequence of FPs focusing on the specific concept of density (Albarracín & Gorgorió, 2018), in which the students successively worked on the following problems:

- FP I: How many people could we fit in the high school courtyard?
- FP IIa: How many people could we fit in the Palau St. Jordi sport pavilion for a concert?
- FP IIb: How many people could we fit in the town hall square during a public protest?
- FP III: How many people could we fit in Plaça Catalunya square during a public protest?
- FP IV: How many trees are there in Central Park?

In this sequence, students developed models and strategies to estimate the number of objects in different two-dimensional enclosed areas of different (i) sizes; (ii) contexts; and (iii) proximities to own personal experiences (Albarracín & Gorgorió, 2018).

This last example illustrates how FPs, in a designed sequence, can connect mathematical content explored in different contexts and integrate these into a more widely applicable knowledge in the form of a generalised model. This model can then be further connected to other mathematical ideas (abstract or concrete) and integrated into a more complex web of knowledge (Ärlebäck & Albarracín, 2019). As FPs are open, they allow for multiple different solutions using different strategies and drawing on different content (Ärlebäck, 2009; Carlson, 1997). In a classroom setting, sharing students' solutions will highlight the variety of approaches that can be taken, which explicit mathematical content comes to the fore, and how these can be combined to produce a more efficient and correct solution. In this way, in addition to eliciting content, FPs can facilitate the connection and integration of intra-mathematical content.

Although an FP is generally a focused and specific question, the openness of FPs *induces* problem-posing throughout the whole problem-solving activity. With regard to the intra-mathematical aspects of FPs, the questions posed by the problem-solver focus on *what more specific mathematical content* could be used to solve the FP. Examples of questions in the four phases could be: 'What are the mathematical concepts and structures that I can identify in the FP, and

what is the asked for quantity/quantities?' (*orientation*); 'What am I wondering about in relation to the requested quantity/quantities, and to what other quantities are they related?' (*connection*); 'How can I represent the identified quantity/quantities that are important for solving the FP?' (*generation*); and 'Are the identified sub-problems and the chosen mathematical construct used to solve them correct or optimal in order to solve the FP?' (*reflection*).

3.2 Fermi problem as extra-mathematical connectors and integrators

Notably, FPs have been utilized across all STEM disciplines for various purposes (Ärlebäck & Albarracín, 2019; 2023). This is evidenced in the literature review of the research on FPs in the STEM disciplines presented in Ärlebäck and Albarracín (2019), which analysed 91 educational research journal articles (22 in Science; 12 in Technology & Engineering; 28 in Mathematics; and 8 assorted articles). The review focused on the arguments for using FPs in the STEM disciplines and what the findings of the included studies were, given in four thematic topics: *estimation*; *number sense and beliefs*; *problem solving;* and *modeling.* Here, some brief example of research in Science (S), Technology (T) and Engineering (E) is given (excluding research in Mathematics (M)); STE(M).

3.2.1 Estimation: The STE(M) literature highlights the importance of estimation in different contexts and how FPs can be used to foster estimation skills in engineering (Moore, 1987; Shakerin, 2006) and computer science (Bentley, 1999). The context in which FPs are used for educational purposes varies and includes finding reasonable baseline data/values (which could be validated experimentally), connecting theoretical knowledge with practical applications, and various contexts that require quick decision making (Ärlebäck & Albarracín, 2019).

3.2.2 Number sense and beliefs: The STE(M) research suggests that working with FPs can improve students' understanding of quantities across disciplines, both quantitatively and, more critically qualitatively. The quantitative aspect of number sense has been discussed in science in general by Furjanic and Müller (2001) and in biology in particular by White (2004) and Phillips and Milo (2009). Cordry's (2010) and Morgan's (2017) work, on the other hand, uses the FP to work with students' beliefs and their understanding of the qualitative critical aspects of numbers in context.

3.2.3 Problem solving: Overall, the use of FPs in STE(M) has been shown to enhance students' general reasoning and metacognitive skills. FPs have been shown, for example, to (i) promote flexible thinking and the creation of unique problem-solving methods (Seiwald, 2016); (ii) be a viable tool for supporting students who struggle with the analytical and validation stages of problem solving (Holubova, 2017); and (iii) facilitate learning in the context of technologies such as mobile phone applications and spreadsheets (Tangney and Bray, 2013).

3.2.4 Modelling: The STE(M) research that emphasises the modelling aspects of FPs is typically from the tertiary level in the context of promoting structural thinking, self-monitoring and validation in the solution process. One example is Robinson (2008), who used FPs in Physics to outline the applied problem-solving steps for synthesising a model and using rough estimation as a general method. Another example is Czocher (2016, 2018), while working with engineering students engaged in solving FPs, elicited the complexity of the modelling process for these engineering students and articulated the importance of validation at different stages of modelling.

The above examples from the STE(M) disciplines and the addition of the examples from the Mathematics discipline (M), the cumulative research together, illustrate how FPs can be used as integrators and facilitators in STEM education (Ärlebäck & Albarracín, 2019). The nature of FPs in the STEAM setting differs from single-domain or more restricted FPs, broadly speaking, only in the complexity and number of disciplines that need to be considered to solve the FP adequately. As a consequence, when dealing with FPs in STEAM settings, also *induces* problem-posing throughout the whole problem-solving activity, but now however, the questions posed by the problem-solver focus on *what specific mathematical content*, as well as *what extra-mathematical content*, could be used to solve the FP.

4. Fermi Problem Activity Templates and examples

In the literature review on FPs in various disciplines (Ärlebäck & Albarracín, 2019), FPs as integrators and facilitators in STEM were discussed in relation to learning 21st-century skills (Binkley et al., 2012). Four types of mathematical activities (cf. Rasmussen et al., 2005) were identified that are used to obtain the necessary numerical values to solve the problems in question. These four types of activities (*guesstimation*, *measurement and experimentation*, *looking up data*, and *polling and statistical data collection*) form the core of a framework called *Fermi Problem Activity Templates (FPATs)*, which was developed as a research tool, a tool for task design, and a tool to

support the implementation of FPs in classrooms (Albarracín & Ärlebäck, 2019; Ärlebäck & Albarracín, 2023).

An FPAT is a compact graphical representation of a solution (or intended solution) to a FP, showing both the decomposition of the problem into sub-problems (cf. Figure 2) and how the unknown values of the quantities involved are or will be determined in terms of one of four activities (each represented by its own geometric shape):

(1) *guesstimation* (quickly providing an answer based on prior knowledge and experience) – [an ellipse];

(2) *measurement and experimentation* (working in the laboratory, the field or in the classroom; often using tools) – [a trapezoid];

(3) *looking up data* (in official databases, online or news outlets) – [a trapezoid]; and

(4) *polling or statistical data collection* (using surveys and questionnaires, etc.) – [a hexagon].

The ideas are that, as a research tool, the FPAT can be used as an analytical tool to investigate students' solutions; in task design, the FPAT can facilitate the formulation of content-focused modelling activities linked to official curriculum objectives; and as a tool for teachers to support the implementation of FPs in the classroom by preparing them for what to expect and what material to prepare for their students. To illustrate some of the ideas of the FPAT, I will briefly elaborate on two examples.

4.1 Example 1. An intra-mathematical FPAT: Consider the FP *How many rolls of toilet paper are used in the schools in your country every year?* (Ärlebäck & Albarracín, 2022; 2023). Given this *Toilet Roll Paper Task* we can ask at least two questions: *What does a problem solver have to do to come up with a solution and an answer?* and *What does a student solution look like?* From a design point of view, Figure 3 illustrates an a priori analysis of the *Toilet Roll Paper Task* that explicitly highlights and differentiates between the different sub-problems that students have to engage with in order to solve the problem. However, if Figure 3 is the result of an a posteriori analysis of a (group of) students' work on the *Toilet Roll Paper Task*, then Figure 3 is an easily accessible representation of the solution.

The open-ended nature of an FP means that there are several types of activities that could be considered to obtain the numerical values needed to address the various sub-problems. One could use more conventional methods of estimation based on past experience to obtain information on the number of people in schools (including staff and pupils) and the total

number of school days in a year (*guesstimation*), or one could refer to official school and government records for more precise information (*looking for data*).

An anonymous statistical survey or data collection could be used to determine the number of daily toilet visits per person (*polling or statistical data collection*). This method ensures confidentiality and may provide more accurate data. Given the potential sensitivity of estimating toilet paper use, a *guesstimation* could be used to approximate the average amount of toilet paper used per visit.

To determine the length of paper on a toilet roll, a geometric approach could be used to calculate the total length. Alternatively, as shown in Figure 3, an investigation could be carried out using physical rolls of toilet paper. This could involve unrolling the roll to measure the length directly or using a weighing method to estimate the total length based on the weight of the paper (*measurement and experimentation*).

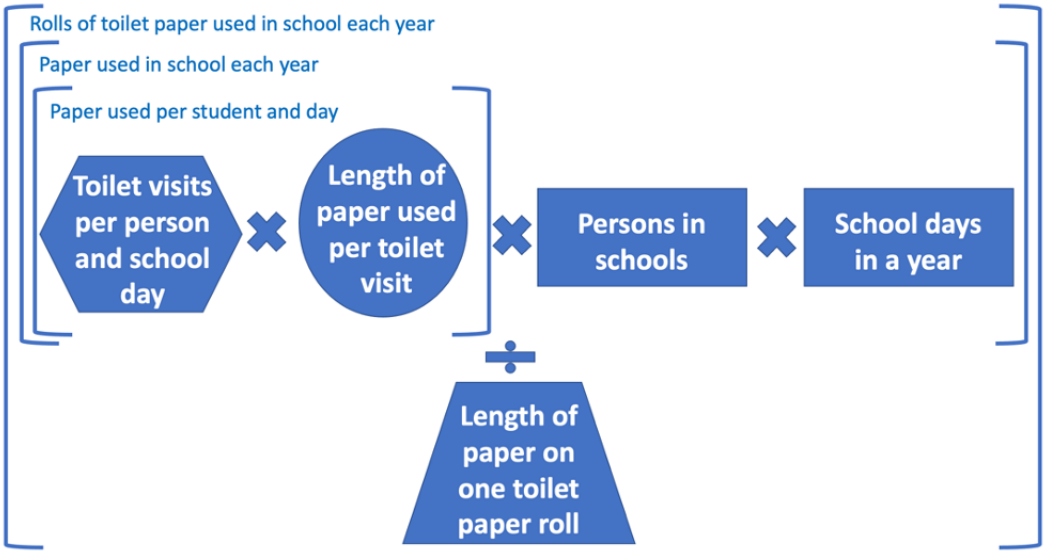

Figure 3. An FPAT of *The Toilet Roll Paper Task*

In summary, note that the brackets in Figure 3 delimit the decomposition of sub-problems and the mathematical operations to be performed to arrive at an answer. As such, the FPAT represents a mathematical model for solving the posed FP. Importantly, an FPAT makes explicit to problem solvers (i) what knowledge they already possess, have mastered, and have readily available; (ii) what is still unknown; and (iii) provides a suggestion of how to proceed to find out the unknowns.

4.2 Example 2. An extra -mathematical FPAT: One example of an FP presented and elaborated in a Physics context is "How much power does an average student use climbing the stairs to the observation deck in the Empire State Building?" (Ärlebäck & Albarracín, 2022, p. 570). Another example can be found in Ärlebäck and Albarracín (2024) in the context of *The tin foil boat challenge* (Lee, 2014). This second example showcase "the potential of FPs and FPATs in facilitating the integration of STEM disciplines as well as highlight the mathematics coming to the fore" (p. 212), and the FP discussed is "*How many coins does it take to sink a tin foil boat made from a sheet of tin foil measuring 30x40 centimetres?*" (p. 213).

An example of an integrated STEAM-oriented FP is the design of an outdoor rain gauge that allows daily and monthly measurements. This type of problem requires not only estimation, but also practical considerations and design thinking, making it a comprehensive educational tool in STEAM education. The problem statement of the FP is *How to design an outdoor rain gauge that allows reading both daily and monthly measurements?* Figure 4 shows a preliminary analysis of the FP and an attempt to highlight the disciplines involved and relevant to the problem using different colours.

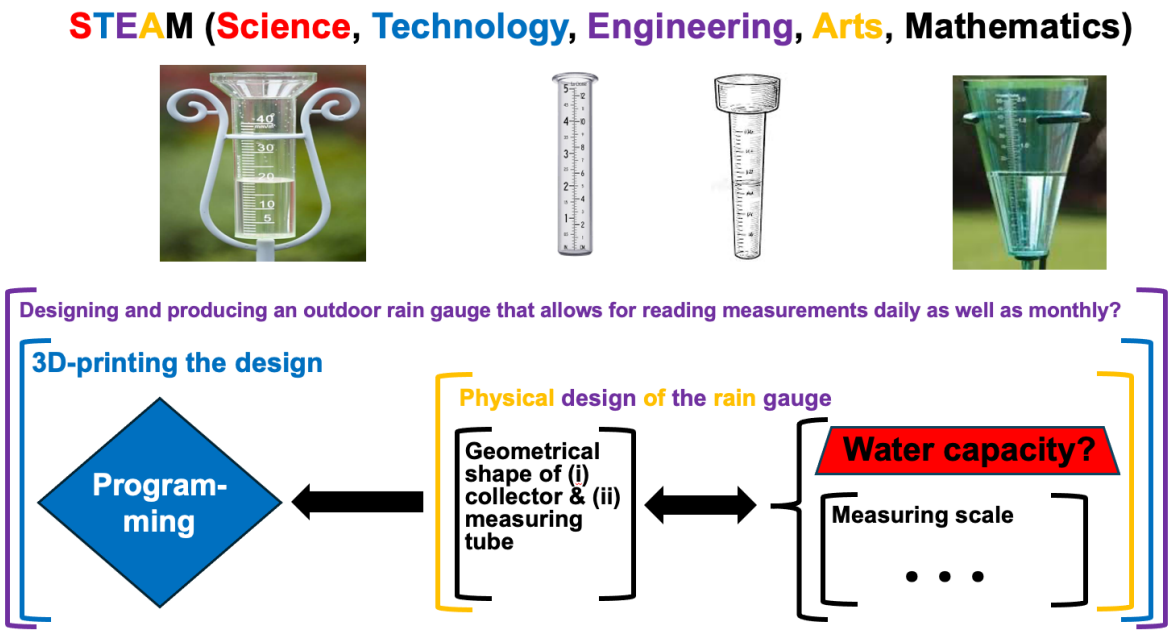

Figure 4. An FPAT of *The Rain Gauge Design Task*

The Rain Gauge Design Task includes the actual production of the rain gauge in terms of 3D-printing the design (Technology), as well as the actual physical design needs to be decided on (Arts) – both of which are new components to the original FPAT framework. Recognising the different STEAM distinctions in the FPAT adds complexity, but colouring the brackets that

demarcate and define the sub-problems potentially provides an easily accessible overview of what needs to be considered in coming up with a solution to the task.

5. Summary and outlook

This paper has discussed recent research focusing on FPs have been discussed as intra- and extra-mathematical content, connectors, and integrators in mathematics and across the STEAM disciplines. Various potential connections to problem-posing (Cai & Hwang, 2020; Cai & Rott, 2024) have been proposed, and the FPAT framework has been highlighted as a tool for research, task design, and classroom implementation of FPs (Albarracín & Ärlebäck, 2019; Ärlebäck & Albarracín, 2023). In addition, examples and recommendations for facilitating the use of Fermi problems were presented. Finally, I now conclude the paper by outlining some potential avenues for future research in this area.

First, a promising venue for future research is to situate problem-posing in relation to FPs and the FPAT framework in STEAM contexts. My initial thinking put forward in this paper regarding the potential connections between Cai and Rott's (2024) heuristic four phase general problem-posing model (*orientation, connection, generation* and *reflection*) and FPs and the FPAT framework suggest possible avenues to explore. One example is thinking and theorising about problem-posing in a broader context - not only emphasising intra-mathematical problem-posing, but also problem-posing in the STEAM disciplines or integrated STEAM education. Another possibility is to investigate what aspects and foci come to the fore in problem-posing in relation to the FPAT when the problem-poser is (i) a researcher using the FPAT as a research tool; (ii) a curriculum designer using the FPAT as a design framework; or (iii) a teacher using the FPAT to support the implementation of FPs in the classroom.

Secondly, the research review found no FP research focusing on the 'A' (Arts) in a STEAM context. Given this gap, several promising research avenues emerge. One area is developing interdisciplinary curricula that incorporate FPs into art projects, such as estimating materials for large-scale installations or the time needed for complex artworks. This approach enhances quantitative reasoning while fostering creativity and problem-solving. Another direction involves investigating the impact of FPs on students' engagement and learning outcomes in art classes. Studies could explore how FPs influence attitudes towards both mathematics and the arts, potentially breaking down perceived barriers. Additionally, research could examine how FPs teach concepts like scale, proportion, and perspective in art, providing an integrative learning experience that bridges quantitative and creative thinking.

Finally, the characteristics of the nature of knowledge in the STEAM disciplines, as well as the nature of inquiry typically practised in each discipline, place new demands on teachers and students in integrated STEM learning settings. Indeed, productive and purposeful teaching and learning in integrated STEM approaches requires an appreciation, coordination and integration of discipline-specific epistemologies and analytical as well as methodological processes (Quinn, Reid, & Garnder, 2020). To clarify the knowns and unknowns for teachers and students in integrated STEM (oriented Fermi) problems, I propose to extend the FPAT framework to:

1. Indicate the primary STEM domain of each sub-problem.
2. Include activities highlighting and emphasising technology (T) and engineering (E).

Visualising the STEM domains of sub-problems will provide an overview of the relevance and interaction of the disciplines and helpthe problem-solvers to identify what is known and unknown in each domain. Different coulors can represent sub-problems from different STEM disciplines, as shown in Figure 4. In addition, the inclusion of activities such as programming (T_P), simulations (T_S), and design (E_D) will enhance the FPAT framework and, based on previous experience, may be an approach worthy further development and exploration.

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