Robust NLS Sensor Localization using MDS Initialization

Viktor Deleskog*, Hans Habberstad*, Gustaf Hendeby*†, David Lindgren*† and Niklas Wahlström†
*Dept. of Sensor & EW Systems, Swedish Defence Research Agency (FOI), SE-581 11, Linköping, Sweden.
Email: firstname.lastname@foi.se
†Dept. Electrical Engineering, Linköping University, SE-581 83 Linköping, Sweden.
Email: {hendeby, david, nikwa}@isy.liu.se

***Abstract*—Before a sensor network can be used for target localization, the locations of the sensors need to be determined. We approach this calibration step by moving a source to distinct positions around the network. At each position, the range to each sensor is measured, and from these range measurements the sensor locations can be estimated by solving a *nonlinear least squares* (NLS) problem. Here we formulate the NLS problem and describe how to robustly initialize it by the use of multidimensional scaling. The method is evaluated on both simulations and real data from an acoustic sensor network. With as few as six source positions, a robust calibration is demonstrated that gives a position error about the same size as the range error. In the acoustic example this RMSE is less than 40 cm.**

I. Introduction

The relative coordinates of the nodes in a passive synchronized sensor network can be determined by encircling the network with a signal source, even when no prior geographical information is available. Based on time of arrival (TOA), the distance between a number of source positions and the sensor positions can be established with well-known filter techniques, provided that the synchronization error in the network is moderated. However, it is not trivial to deduce the coordinates from these measurements, especially in the case when no prior information of the sensor and the source locations are available. This is the anchor-free localization problem.

There are a number of sensor localization problems treated in the literature, see [12] for an overview. Of course, using a *Global Navigation Satellite System* (GNSS) is an appealing solution in many situations, but it may not always give the required accuracy, and it may also be unavailable (indoors) or jammed. Therefore it is highly motivated to study alternative sensor techniques. The most standard sensor measurements used in sensor localization are *time of arrival* (TOA) [2], *time difference of arrival* (TDOA) [6], *angle of arrival* (AOA) [10] and *received signal strength* (RSS) [14]. Sensors equipped with communication capabilities and advanced signal processing offer a opportunity to exchange valuable information between the sensor nodes [9], but with the cost of energy consumption and price.

The problem of deducing coordinates from distance measurements has in many related works been approached with *multidimensional scaling* (MDS), see for instance [12, 14]. MDS does, however, not immediately solve the problem, since it relies on a complete set of distance measurements between all pairs of coordinates. This is not the case in our study, where we have only distance measurements between sources and sensors.

In this work the sensor localization, in a multi-sensor context of stationary sensors, is considered to be a calibration step to be conducted before the sensor network can be used for its dedicated task. Therefore we have full control of the source ourselves — we may design the source signal to promote accurate detection, we may assume that the source is synchronized with the sensors, and we may also choose the source positions to facilitate the calibration. The signal detection may be distributed in the network, but the localization algorithm proposed here is centralized, so the sensors need to have means to transmit the detections to a fusion node where the coordinates are estimated.

We formulate the sensor localization as a *nonlinear least squares* (NLS) problem. The main contribution of the work is the way the NLS solver is initialized, which is a critical task since the NLS objective function has many unknowns, basically all source and sensor locations, and to that end is non-convex. The initialization is based on a number of combined MDS solutions. In [2], only the sensor positions are estimated with MDS, but here we simultaneously estimate the source and the sensor positions.

The background to the work is acoustic sensor networks for target localization and tracking. Examples of earlier contributions in this area are related to sniper localization, see [7], and Doppler-based localization, see [8]. Of course there are numerous other applications where the nodes of the sensor network need to be determined, and the methods presented herein are not at all restricted to acoustic sensors, but applicable to any type of sensor network that supports range measurements. More examples are given in [13, 15].

The paper is organized as follows. In Section II, the problem to localize the sensors and sources is presented. In Section III we cover how to initialize the NLS sensor localization algorithm using MDS. With the presented initialization the NLS problem is solved in Section IV. In Section V the impact of measurements errors are investigated by simulations, and the robustness is tested through real measurements. Finally, Section VI presents the paper conclusions.

II. Problem description

Consider a synchronized sensor network with N stationary microphones at unknown locations, $\mathrm{s} = [s_1 \quad s_2 \quad \dots \quad s_N]$, and a sound source that is moved around outside the convex hull of the network and transmitting a sound pulse from K unknown locations, $\mathbf{x} = [x_1 \quad x_2 \quad \dots \quad x_K]$. Using TOA the sensors measure the distance to the source as

$$y_{k,n} = h(x_k, s_n) + e_{k,n} \tag{1a}$$
$$h(x_k, s_n) = \|x_k - s_n\|_2 \tag{1b}$$
$$e_{k,n} \sim \mathcal{N}(0, \sigma_{k,n}^2), \tag{1c}$$

where the speed of sound in air c is considered a known parameter. The measurement error, $e_{k,n}$, is considered to be white Gaussian noise with known variance $\sigma_{k,n}^2 = \sigma^2$. The problem to estimate the sensor and source locations can be formulated as an NLS optimization problem

$$(\hat{\mathbf{x}}, \hat{\mathbf{s}}) = \arg\min_{\mathbf{x},\mathbf{s}} V(\mathbf{x}, \mathbf{s}) \tag{2a}$$

with the cost function equal to

$$\begin{aligned} V(\mathbf{x}, \mathbf{s}) &= \sum_{k=1}^{K} \|\mathbf{y}_k - \mathbf{h}(x_k, \mathbf{s})\|_2^2 \\ &= \big(\mathbf{Y} - \mathbf{H}(\mathbf{x}, \mathbf{s})\big)^T \big(\mathbf{Y} - \mathbf{H}(\mathbf{x}, \mathbf{s})\big), \end{aligned} \tag{2b}$$

which is the sum of the squared residuals where

$$\mathbf{y}_k = [y_{k,1} \quad y_{k,2} \quad \dots \quad y_{k,N}]^T \tag{3a}$$
$$\mathbf{Y} = \begin{bmatrix}\mathbf{y}_1^T & \mathbf{y}_2^T & \dots & \mathbf{y}_K^T\end{bmatrix}^T \tag{3b}$$
$$\mathbf{h}(x_k, \mathbf{s}) = [h(x_k, s_1) \quad h(x_k, s_2) \quad \dots \quad h(x_k, s_N)]^T \tag{3c}$$
$$\mathbf{H}(\mathbf{x}, \mathbf{s}) = \begin{bmatrix}\mathbf{h}(x_1, \mathbf{s})^T & \mathbf{h}(x_2, \mathbf{s})^T & \dots & \mathbf{h}(x_K, \mathbf{s})^T\end{bmatrix}^T. \tag{3d}$$

Note that minimizing the quadratic cost function $V(\mathbf{x}, \mathbf{s})$ is equivalent to maximizing the likelihood of the obtained measurements. This is one motivation for the chosen problem formulation.

In this paper the NLS problem will be solved using the Gauss-Newton method initialized by a novel version of MDS tailored to suit the specific problem at hand.

III. Initialization

To the best of our knowledge, (2a) can only be solved numerically. The Gauss-Newton method [11] is one such numerical method, which is known for its fast convergence. However, it must be initialized close to the optimum to be guaranteed to converge. MDS provides an option to obtain an initial estimate, which has previously been used in connection to sensor networks [14]. Another common use of MDS is to visualize high-dimensional data in a low-dimensional geometric representation, often in two or three dimensions. This section presents the MDS method, before showing how MDS can be used to initialize a more advanced NLS method to solve the calibration problem.

A. *Multidimensional scaling (MDS)*

MDS is a method to provide relative coordinates of given points based on information about the relative distances between the points. Denote with $D \in \mathbb{R}^{N \times N}$ the *distance matrix*, with the elements

$$d_{ij} = \|s_i - s_j\|_2, \tag{4}$$

the Euclidean distance between sensor i and j. The MDS theory states that the matrix D is sufficient to exactly reconstruct the coordinates of the points $\mathbf{s}$ up to an isometric transformation [14]. Consider the matrix

$$B = \mathbf{s}^T \mathbf{s} \in \mathbb{R}^{N \times N}, \tag{5}$$

which relates to the distances in D as

$$\begin{aligned} d_{ij}^2 &= \|s_i - s_j\|_2^2 = (s_i - s_j)^T (s_i - s_j) \\ &= s_i^T s_i - 2 s_i^T s_j + s_j^T s_j = b_{ii} - 2b_{ij} + b_{jj}, \end{aligned} \tag{6}$$

where b_{ij} are the elements of B. Given D, each element b_{ij} can be calculated as

$$\begin{aligned} b_{ij} = -\frac{1}{2}\Bigg(d_{ij}^2 - \underbrace{\frac{1}{N}\sum_{g=1}^{N} d_{ig}^2}_{\text{Row mean}} - \underbrace{\frac{1}{N}\sum_{g=1}^{N} d_{gj}^2}_{\text{Column mean}} \\ + \underbrace{\frac{1}{N^2}\sum_{g=1}^{N}\sum_{h=1}^{N} d_{gh}^2}_{\text{Grand mean}} \Bigg). \end{aligned} \tag{7}$$

Assume that the distances in D are generated by points in an r-dimensional space where $r \le N$. (In sensor networks setting, the sensors are confined to two or three dimensions.) The following procedure can then be used to extract their relative positions using a (truncated) *singular value decomposition* (SVD).

Compute the truncated SVD of B to obtain a rank r approximation of B [4],

$$B = U \Sigma V^T = U \Sigma U^T \approx U_r \Sigma_r U_r^T, \tag{8}$$

where $U = V$ are orthogonal matrices (equality follows since B is assumed to be symmetric and positive semidefinite), and Σ is diagonal with the singular values $\gamma_1^2 > \gamma_2^2 > \dots > \gamma_N^2 > 0$ on the diagonal. The best r-rank approximation of B is then given by the first r singular values, *i.e.*, Σ_r comprises the first r singular values and U_r the first r columns of U.

An estimate of the coordinates of the points is then given by

$$\hat{\mathbf{s}}^{\text{MDS}} = (U_r \Sigma_r^{1/2})^T = \Sigma_r^{1/2} U_r^T. \tag{9}$$

MDS is summarized in Algorithm 1.

Algorithm 1 Multidimensional Scaling (MDS)

1: Form the distance matrix D with distances between the given points.
2: Calculate B using (7).
3: Compute the truncated SVD of B by keeping the first r singular values, $B \approx U_r \Sigma_r U_r^T$.
4: The estimate of the coordinates of the points are $\hat{\mathbf{s}}^{\text{MDS}} = \Sigma_r^{1/2} U_r^T$.

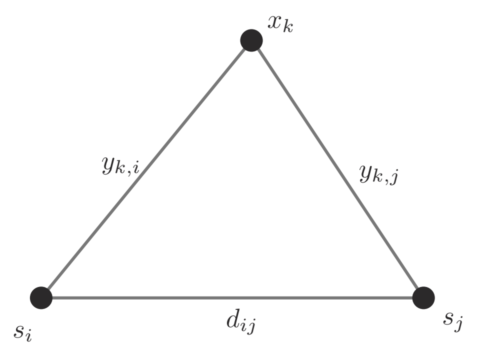

Fig. 1. Illustration of the reverse triangle inequality to approximate the distance d_{ij}, given the measurements $y_{k,i}$ and $y_{k,j}$.

B. *MDS for NLS initialization*

The problem description states that a sound source moves outside the network transmitting sound pulses at different locations x_k. To apply MDS the distance matrix D is needed. It is, however, not readily available because the measurements (1) do not measure the needed inter-sensor distances directly. Instead the elements d_{ij} in D must be approximated. This can be done using the reverse triangle inequality

$$\big|\|x_k - s_i\|_2 - \|x_k - s_j\|_2\big| \leq \|s_i - s_j\|_2, \qquad (10)$$

as illustrated in Fig. 1. It should be obvious from (10) that the quality of the approximated distance depends on the source location x_k. Equality in (10) is obtained with the sound source located on the extension of a straight line passing through the two sensors, and the worst case estimate 0 is obtained with the source equidistant from both sensors.

By ensuring that at least one sound source location lines up with each sensor pair, it is straightforward to show that D can be obtained as

$$\hat{d}_{ij} = \max_{k=1,\dots,K} |y_{k,i} - y_{k,j}|, \qquad (11)$$

if the measurement noise is ignored. However, this is not practically possible, as the sensor positions are assumed unknown. A practical approximation is to spread the source locations evenly around the sensor network,

$$\phi_k = \big(\tfrac{k-1}{K} + \tfrac{1+(-1)^k}{2}\big)\pi \qquad (12a)$$

$$x_k = \begin{bmatrix}\rho\cos\phi_k \\ \rho\sin\phi_k\end{bmatrix}, \qquad (12b)$$

where ρ is large enough for the sources to encircle the sensor network. This way, the offsets from the optimal source locations are minimized, and it should be possible to quantify the worst case behavior. As $K \to \infty$ there will always be a source aligned with each sensor pair, and the error goes to 0. It has been observed that measurements from as few as 4–5 source locations are sufficient to obtain an acceptable approximation of D. Define $\hat{D}$ as the estimated distance matrix, where each element is calculated as in (11). With the estimate $\hat{D}$ the unknown sensor positions can be estimated by MDS to obtain $\hat{\mathbf{s}}^{\text{MDS}}$.

Still, the NLS problem cannot be initialized as the source locations x_k are unknown. This is a drawback of how MDS

Algorithm 2 MDS Initialization for NLS

Compute the distance matrix $\hat{D}$ using (11).
Compute $\hat{\mathbf{s}}^{\text{MDS}}$ with MDS (Algorithm 1) using $\hat{D}$.
For each source x_k:
    Form the extended distance matrix $\hat{D}_k$ as in (13).
    Compute $\hat{x}_k^{\text{MDS},k}$ and $\hat{\mathbf{s}}^{\text{MDS},k}$ with MDS using $\hat{D}_k$.
    Align $\hat{\mathbf{s}}_k^{\text{MDS},k}$ to $\hat{\mathbf{s}}^{\text{MDS}}$ using (14) and compute $\hat{x}_k^{\text{MDS}}$
    using (15).
    Append $\hat{x}_k^{\text{MDS}}$ to $\hat{\mathbf{x}}^{\text{MDS}}$.
The sensor locations are now found in $\hat{\mathbf{s}}^{\text{MDS}}$ and the source locations in $\hat{\mathbf{x}}^{\text{MDS}}$.

commonly is applied in sensor localization applications. A solution would be to also include the source locations in the D matrix, and estimate them simultaneously using MDS. This would require both the distance between the sensors and the sources, $\|x_k - s_n\|_2$, and the distance between the sources $\|x_k - x_l\|_2$. The former distance is obtained directly from the measurements, whereas there is no way to obtain the source to source distances. This makes this formulation infeasible.

Instead, estimate the source locations one at the time. To estimate the location of source k form

$$\hat{D}_k = \begin{bmatrix} & \hat{D} & & y_{k,1} \\ & & & \vdots \\ & & & y_{k,N} \\ y_{k,1} & \dots & y_{k,N} & 0 \end{bmatrix} \qquad (13)$$

where $y_{k,n}$ approximates $\|x_k - s_n\|_2$. The extended distance matrix $\hat{D}_k$ can directly be input to MDS to estimate the source location, $\hat{x}^{\text{MDS},k}$, and the sensor positions, $\hat{\mathbf{s}}^{\text{MDS},k}$. Since MDS is invariant under isometric transformations, $\hat{\mathbf{s}}^{\text{MDS},k}$ and $\hat{\mathbf{s}}^{\text{MDS}}$ cannot be directly compared. By finding the isometric transformation (R, t) between them, the estimated source location can be correctly aligned with the $\hat{\mathbf{s}}^{\text{MDS}}$. The transformation can be found as the solution to

$$(\hat{R}, \hat{t}) = \arg\min_{R,t} \sum_{n=1}^{N} \big\|\hat{s}_n^{\text{MDS}} - (R\hat{s}_n^{\text{MDS},k} + t)\big\|_2^2, \qquad (14)$$

subject to (R, T) describing an isometric transformation, which find the best transformation in least squares sense. An SVD can be used to solve (14), as described in [1]. The source aligned to the previously obtained sensor locations is then obtained as

$$\hat{x}_k^{\text{MDS}} = \hat{R}\hat{x}_k^{\text{MDS},k} + \hat{t}. \qquad (15)$$

The above procedure can be repeated for each source location to obtain a consistent initial estimate of sensor and source locations. The algorithm is summarized in Algorithm 2. Now the MDS based initial estimate can be used to apply the Gauss-Newton method to solve the problem in (2a).

IV. Nonlinear least squares solution

Using the initial estimates of the source and sensor locations, as derived in Sec. III, the NLS problem in (2a) can now be solved with standard numerical methods such as the Gauss-Newton method [11]. To apply the Gauss-Newton method the

Jacobian, $J(\mathbf{x},\mathbf{s})$, of the residual,

$$J(\mathbf{x},\mathbf{s}) = \frac{\partial}{\partial(\mathbf{x},\mathbf{s})}\big(\mathbf{Y} - \mathbf{H}(\mathbf{x},\mathbf{s})\big) = -\frac{\partial}{\partial(\mathbf{x},\mathbf{s})}\mathbf{H}(\mathbf{x},\mathbf{s}), \quad (16)$$

is required. The Jacobian can also be used to approximate the uncertainty in the estimates [5]

$$\mathrm{Var}\begin{bmatrix}\hat{\mathbf{x}}^{\mathrm{NLS}}\\ \hat{\mathbf{s}}^{\mathrm{NLS}}\end{bmatrix} \approx \sigma^2\big(J(\hat{\mathbf{x}}^{\mathrm{NLS}},\hat{\mathbf{s}}^{\mathrm{NLS}})^T J(\hat{\mathbf{x}}^{\mathrm{NLS}},\hat{\mathbf{s}}^{\mathrm{NLS}})\big)^{-1}. \quad (17)$$

The variance of each individual source and sensor location can be found on the block diagonal of $\mathbf{P}^{\mathrm{NLS}}$.

V. Numerical experiments

In this section the suggested method to calibrate a sensor network is evaluated. The evaluation uses both numerical simulations and real experimental measurements [3] and considers only 2-dimensional sensor network configurations, *i.e.* $r = 2$. The performance metric used is the localization *root mean square error* (RMSE), which is easy to interpret as it indicates how far off the estimated positions are from the true positions. The RMSE for the sensor positions is computed for both the MDS solution and the NLS solution. The position RMSE can not be directly computed, due to the isometric invariance property of the MDS solution, hence the RMSE has been formulated as

$$\hat{P}_{\hat{\mathbf{s}}} = \min_{R,t}\sqrt{\tfrac{1}{N}\textstyle\sum_{n=1}^{N}\|s_n - (R\hat{s}_n + t)\|_2^2} \quad (18a)$$

$$\hat{P}_{\hat{\mathbf{x}}} = \min_{R,t}\sqrt{\tfrac{1}{K}\textstyle\sum_{k=1}^{K}\|x_k - (R\hat{x}_k + t)\|_2^2}, \quad (18b)$$

subject to (R,t) describing an isometric transformation. Estimates of R and t can be obtained as described in Sec. III-B. This modified RMSE definition makes the RMSE values independent of the unknown isometric transformation.

Two key parameters that affects the localization RMSE are

(i) The measurement error in the sensors, σ^2.
(ii) The source locations $x_1, \dots, x_K$.

The last parameter is crucial because the estimated distance matrix $\hat{D}$ is highly influenced by where the sources are located.

A. Simulated data

To demonstrate the properties of the suggested method data has been generated from randomized sensor networks with sensors uniformly spread around them according to (12). Fig. 2 shows one example of such a sensor network. Sensors are marked with crosses and source locations with squares throughout this section. Multiple network realizations are created, during *Monte Carlo* (MC) simulations, to get reliable significant results. A unique network was realized for each MC run. Sensor measurements are generated by adding independent samples of a Gaussian distribution with zero mean and standard deviation $\sigma = 30\,\mathrm{cm}$ to the measured distances, according to (1). All parameters used in the simulation data are given in Table I.

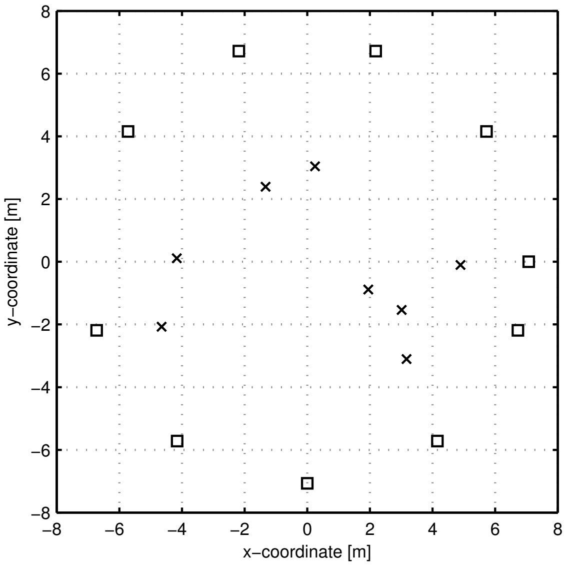

Fig. 2. An example of a randomized network with 8 sensors (crosses) and 10 source locations (squares)

TABLE I. Parameters for the simulated data.

Parameter	Value
Sensors, N	8
Source locations, K	10
Area, A	$10\,\mathrm{m} \times 10\,\mathrm{m}$
Measurement error std. dev., σ	$0.30\,\mathrm{m}$
Monte Carlo runs	500
Radius of source locations, ρ	$7\,\mathrm{m}$

B. Real data

The method has also been tested on real data collected outdoors at a field trial with high quality recording equipment capable of sampling multiple channels synchronously. In this case the sound source emitted a linear chirp pulse for the sensors to detect. At each location the sound was played 10 times, then the average of the measurement was used in the consecutive steps, and the measurement noise was estimated based on the 10 measurements. The TOA estimates were obtained using a simple cross-correlation detector. More detailed information about the data and how it was collected can be found in [3].

Data was recorded from two network configurations. In the first configuration the sensors were densely deployed and the source were moved to locations close to the sensors to achieve good *signal-to-noise ratio* (SNR). In the second configuration the sensors were deployed at greater distances, and the source was moved to locations further away from the sensors to record measurements with lower SNR. Fig. 3 and 4 show the first and second configuration, respectively. The configurations and recording parameters are specified in Table II. The real data is evaluated with respect to the number of locations the source was moved to, K. The results are computed from 10 different network configurations where the source locations are picked randomly, from the available source locations, to get a unique source configuration in each case.

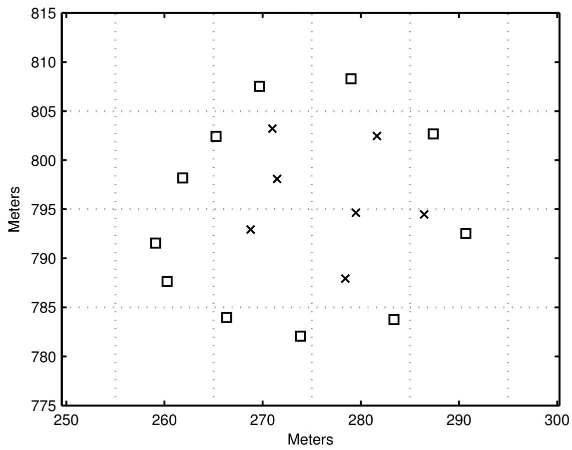

Fig. 3. The first outdoor configuration with 7 sensors (crosses) and 11 source locations (squares).

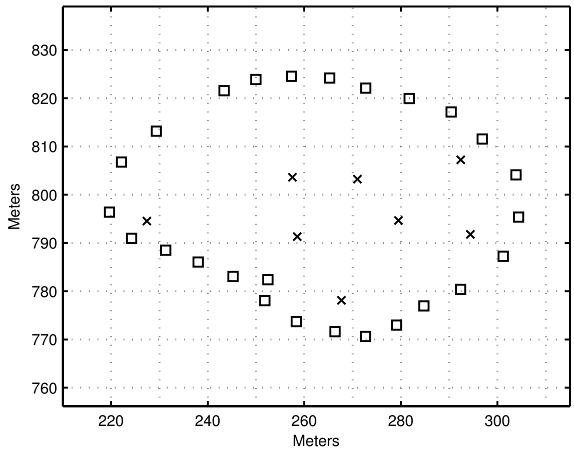

Fig. 4. The second outdoor configuration with with 8 sensors (crosses) and 26 source locations (squares).

C. Results

First consider the results obtained using the simulated data. As an example of the estimated sensor locations, consider the sensor network realization in Fig. 2. The final NLS calibration result is shown in Fig. 5, where the uncertainties in the estimates are indicated with ellipses (representing 95% confidence bounds). The localization errors are illustrated with lines drawn from estimated positions to true positions, where true positions are illustrated with circles. The ellipses in Fig. 5 indicate that it is easier to estimate the sensor positions than the source positions. This is natural since there are K measurements for each sensor obtained with K different sound source locations, and only N measurements all from the same source location for each source. The localization errors confirm that the estimated confidence bounds are correct.

TABLE II. CONFIGURATION PARAMETERS FOR THE REAL DATA.

Parameter	Configuration	
	First	Second
Sensors, N	7	8
Source spots, K	11	26
Mean sensor-to-sensor distance, d_{ij}	10 m	30 m
Signal sound level @ 750–1250 Hz	106 dB SPL	
Background noise level @ 750–1250 Hz	33 dB SPL	

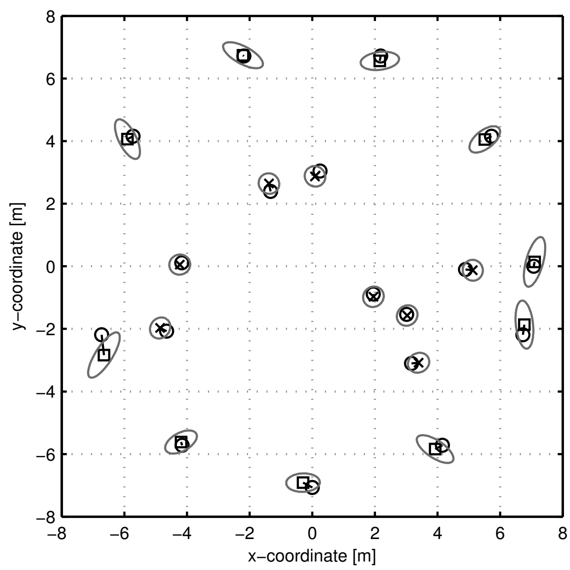

Fig. 5. Localization results from data generated with a randomized network. Estimated sensors (crosses) and sources (squares) with 95 % confidence bounds illustrated with ellipses. The localization errors are illustrated with lines to true positions (circles).

The impact of the measurement error is evaluated in Fig. 6 based on 500 MC simulations with 10 source locations. The MDS solution does not improve when standard deviation of the measurement drops below 10 cm. It can hence be assumed that the approximation used to obtain $\hat{D}$ dominates in scenarios with low measurement error. The NLS solution does not use the approximated D matrix, and is close to linear in the noise level, considerably improving the estimates especially for low measurement noise where the $\hat{D}$ approximation affects the MDS estimates the most.

Fig. 7 shows results from varying the number of source locations that are used for the estimate, using 500 MC simulations and $\sigma = 30$ cm. The figure shows considerable improvement when at least three source locations are used in both the MDS and NLS estimates. Furthermore, the MDS seems to stabilize at an error of 30 cm, which is the same level as the measurement noise. This can probably be explained by the way only the largest estimated distance between each two sensors are use in $\hat{D}$, hence not allowing for more than one measurement to be used to average the error.

Moving on to the results obtained using the real data. From the acquired data the measurement error was computed, as shown in Fig. 8, and shown to be approximately 33 cm in the range 10–90 m. Fig. 9 shows the localization RMSE with the first configuration for both the MDS and NLS solution as the number of source locations used increase. When all source locations are used the NLS solution attains approximately an RMSE of 30 cm, and the MDS solution an RMSE of approximately 50 cm. The overall behavior is the one predicted by the simulations; however, the results are slightly worse which

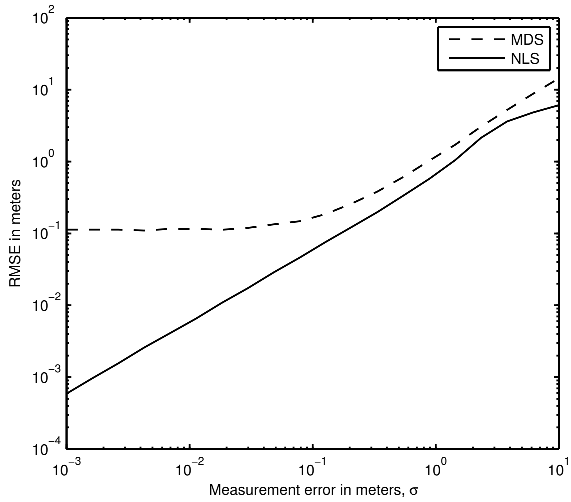

Fig. 6. Sensor localization RMSE for the MDS and NLS solutions as the standard deviation of the measurement error is increased using simulated data.

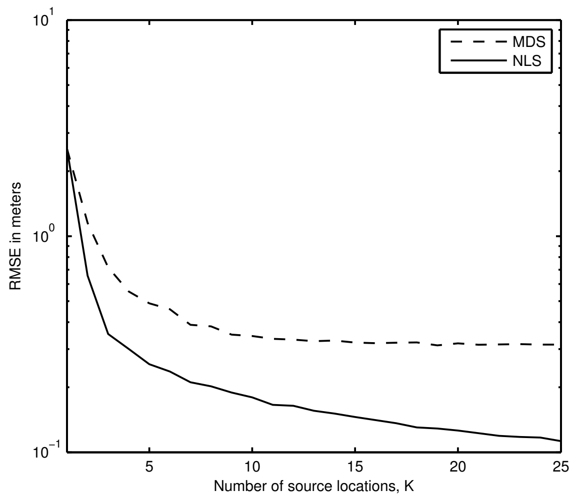

Fig. 7. Sensor localization RMSE for the MDS and NLS solutions when the source is moved to more locations using simulated data.

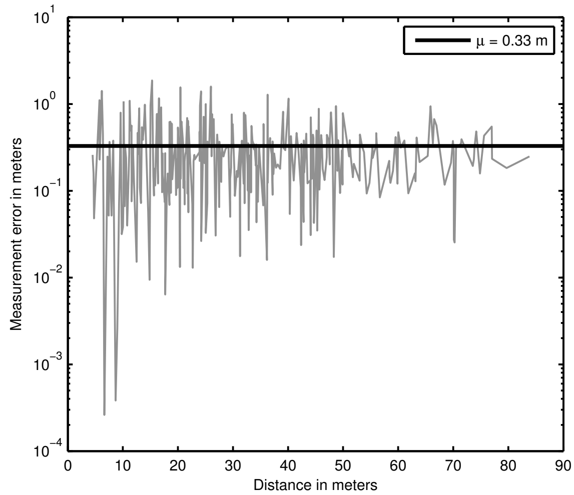

Fig. 8. Measured measurement error of real data collected outdoors. The mean measurement error is $0.33\,\mathrm{m}$.

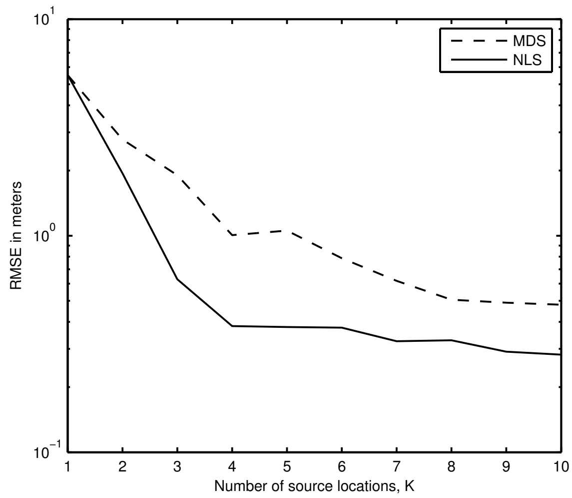

Fig. 9. Sensor localization RMSE for real data from the first configuration as the number of source locations is increases.

indicates other sources of errors than those modeled. The results from the second configuration in Fig. 10 collaborates this.

VI. Conclusion

We have described how the coordinates of passive sensors in a sensor network can be determined by using range measurements to a collaborative source that is moved around the network. The localization is formulated as a *nonlinear least squares* (NLS) problem, where the unknowns are the coordinates of both the sensors and the distinct source positions. The NLS optimum can be pursued by, for instance, the Gauss-Newton minimizer, but the success relies on a good initial estimate. We have proposed a new method that solves a number of *multidimensional scaling* (MDS) problems to find a robust and efficient initial estimate. The algorithm is evaluated on both simulated and real acoustic data for a network with 7–8 sensors. Efficient localization is demonstrated when the number of source positions is six or more. Then the localization error is roughly the same as the range error. For the acoustic dataset the localization RMSE is less than 40 cm. Future studies will investigate the robustness of the proposed algorithm, and to develop methods to efficiently mitigate degeneration due to multi-path propagation.

Acknowledgment

The work was supported by the project Cooperative Localization (CoopLoc) funded by Swedish Foundation for Strategic Research (SSF) and the Swedish strategic research center Security Link. It was also supported by research projects at the Swedish Defence Research Agency (FOI), projects that are funded by the Swedish Armed Forces.

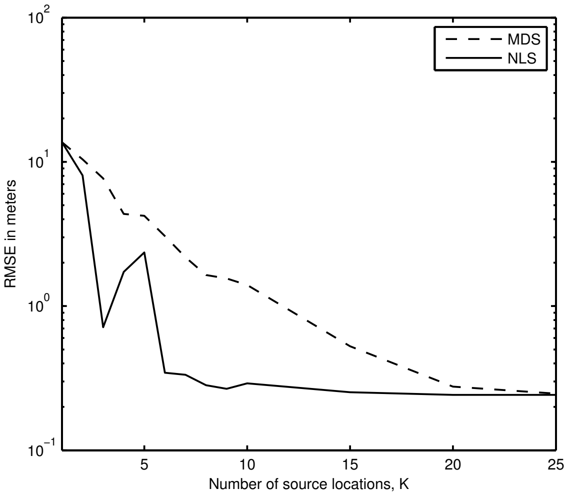

Fig. 10. Sensor localization RMSE for real data from the second configuration as the number of source locations is increases.

References

[1] K. S. Arun, T. S. Huang, and S. D. Blostein. Least-squares fitting of two 3-D point sets. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 9(5):698–700, Sept. 1987.

[2] R. Biswas and S. Thrun. A distributed approach to passive localization for sensor networks. In *Proceedings of the National Conference on Artificial Intelligence*, volume 20, Pittsburgh, PA, July 2005.

[3] V. Deleskog, H. Habberstad, G. Hendeby, and D. Lindgren. Acoustic and visual sensor network measurements in Lilla Gåra 2012. Technical Report MEMO4833, The Swedish Defence Research Agency, FOI, Linköping, Sweden, To appear 2014, Preliminary version available upon request. URL http://www.foi.se.

[4] C. Eckart and G. Young. The approximation of one matrix by another of lower rank. *Psychometrika*, 1(3): 211–218, Sept. 1936.

[5] F. Gustafsson. *Statistical Sensor Fusion*. Studentlitteratur, 2 edition, 2012.

[6] T. Janson, C. Schindelhauer, and J. Wendeberg. Self-localization application for iPhone using only ambient sound signals. In *2010 International Conference on Indoor Positioning and Indoor Navigation (IPIN)*, pages 1–10, ETH Zurich, Switzerland, Sept. 2010.

[7] D. Lindgren, O. Wilsson, F. Gustafsson, and H. Habberstad. Shooter localization in wireless microphone networks. *EURASIP Journal on Advances in Signal Processing*, 2010(1), July 2010.

[8] D. Lindgren, M. B. Guldogan, F. Gustafsson, H. Habberstad, and G. Hendeby. Acoustic source localization in a network of Doppler shift sensors. In *16th International Conference on Information Fusion*, Istanbul, Turkey, July 2013.

[9] D. Moore, J. Leonard, R. Daniela, and S. Teller. Robust distributed network localization with noisy range measurements. In *Proceedings of the 2nd international conference on Embedded networked sensor systems*, pages 50–61, Baltimore, MD, Nov. 2004.

[10] R. L. Moses, D. Krishnamurthy, and R. M. Patterson. A self-localization method for wireless sensor networks. *EURASIP Journal on Advances in Signal Processing*, 2003(4):348–358, Mar. 2003.

[11] J. Nocedal and S. Wright. *Numerical optimization*. Springer, New York, 1999.

[12] N. Patwari, J. N. Ash, S. Kyperountas, A. O. Hero, R. L. Moses, and N. S. Correal. Locating the nodes: cooperative localization in wireless sensor networks. *IEEE Signal Processing Magazine*, 22(4):54–69, July 2005.

[13] A. Savvides, M. Srivastava, L. Girod, and D. Estrin. *Localization in Sensor Networks*, chapter 15, pages 327–349. Springer, 2004.

[14] Y. Shang, W. Rumi, Y. Zhang, and M. Fromherz. Localization from connectivity in sensor networks. *IEEE Transactions on Parallel and Distributed Systems*, 15(11): 961–974, Nov. 2004.

[15] T. Wark, C. Crossman, W. Hu, Y. Guo, P. Valencia, P. Sikka, P. Corke, C. Lee, J. Henshall, K. Prayaga, J. O'Grady, M. Reed, and A. Fisher. The design and evaluation of a mobile sensor/actuator network for autonomous animal control. In *Proceedings of the 6th International Conference on Information Processing in Sensor Networks*, pages 206–215, Cambridge, MA, Apr. 2007.